Refining Fitts' law models for bivariate pointing

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ABSTRACT
We investigate bivariate pointing in light of the recent progress in the modeling of univariate pointing. Unlike previous studies, we focus on the effect of target shape (width and height ratio) on pointing performance, particularly when such a ratio is between 1 and 2. Results showed unequal impact of amplitude and directional constraints, with the former dominating the latter. Investigating models based on the notion of weighted ℓ_p norm, we found that our empirical findings were best captured by an Euclidean model with one free weight. This model significantly outperforms the best model to date.

Keywords
Pointing, Fitts' law, input, human performance

INTRODUCTION
Fitts' law [4] is often used as a model for pointing actions in user interfaces. It predicts that the time T to acquire a graphical target depends on its width W and its distance D to the cursor (Figure 1.a) according to the relation:

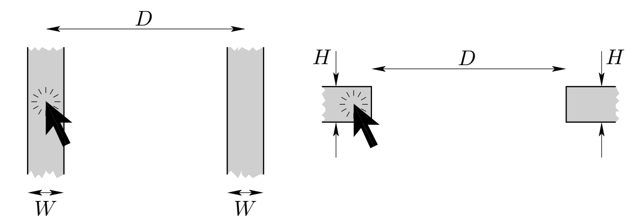
$$T = a + b\,\log_2\left(\frac{D}{W} + 1\right) \quad (1)$$

where a and b are empirically-determined constants. The logarithm term is the index of difficulty (ID) of the task.

Despite Fitts' law success as one of the very few quantitative models applicable to HCI tasks and the great number of studies related to Fitts' law existing in the literature (see [8]), some basic questions regarding Fitts' law have not been satisfactorily addressed. One of them is how Fitts' law should be formulated when the pointed target is two dimensional (2D).

Fitts' law is inherently a 1D model. In experimental studies, it is typical to set the width of the pointing targets to a set of controlled values, but leave the target height (H) practically at infinity. The only constraint on the movement is hence one-dimensional and collinear to the motion direction, with well defined task parameters (D, W).

Practically, meaningful targets are of course 2D. A special class of 2D targets is objects with equal width and height, such as a square or a circle. Such targets have also been used in Fitts' law studies [7], and the usual practice is to take the diameter of the inscribed circle as the W in Fitts' law. In fact, two of the three tasks (peg in hole and disk transfer) in Fitts' original study [4] involved circle-shaped targets.



(a) ***AP*** — Pointing with amplitude constraint

(b) ***DP*** — Pointing with directional constraint

Figure 1: The *AP* and *DP* tasks

Problem arises when the 2D target is elongated, i.e. with unequal width and height[1]. The 1D model applies when H is infinity; it also "works" when $H = W$. The questionable cases are when H approaches W. Intuitively, when H is much greater than W (e.g. $H > 8W$), it does not matter (movement time is only determined by W) because the natural spread of the hit points in the vertical dimension would not exceed such a range. What if H approaches $3W$, $2W$, or even below $2W$? It is conceivable that the additional constraint in the H dimension would make the task harder to perform when H decreases toward the value of W.

Conceptually, we can view a pointing task as controlling both amplitude and directional error. The former is constrained by W and defines an "amplitude pointing" (*AP*) task; the latter is constrained by H and defines a "directional pointing" (*DP*) task[2]. In other words, pointing is a bivariate process.

Stemmed from the need to model and compare goal-passing tasks with pointing tasks, recent work [2] shed some light on the question of directional-error control. The study took the other extreme case of pointing by controlling H and having W infinite (Figure 1.b), such that the only task constraint was directional. It was found that pointing with directional constraint, also follows Fitts' law, with the target height H used in place of W. The study also found that, for the same ID, directional pointing takes less time than amplitude pointing.

[1] In this paper, the width W is always laid in horizontal dimension and the height H in vertical dimension. More generally, W extends in movement direction and H extends orthogonally to movement direction.

[2] The *AP* and *DP* tasks correspond to the *CP* and *OP* tasks in [2]. We herein use different names as *CP* vs. *OP* could be ambiguously interpreted.

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This is a critical step in the understanding of bivariate pointing: for the first time, we isolated the effect of target height on acquisition time, independently of target width.

What is unresolved, however, is the interaction between target height and width. We wondered earlier how movement time would change when H decreases towards the value of W. Symmetrically, if W is greater than H, how does movement time change when W decreases from infinity towards the value of H? More formally, is it possible to find a model comparable to the original Fitts' law that would model the time to acquire a target of finite width and height (Figure 2)?

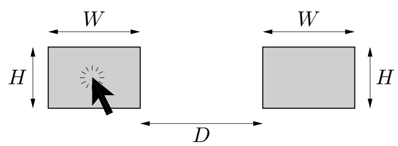


Figure 2: Definition of the task parameters

Although posed from different angles, 2D Fitts' law is not a new topic. In the rest of the paper, we briefly review related literature and summarize the different models proposed so far. We then discuss the limitations of each model and propose new models that are more compatible with the current knowledge of pointing actions.

LITERATURE OVERVIEW

The very first study of two-dimensional pointing seems due to Crossman [3]. Reported by Hoffmann [5], Crossman found that "the vertical height [...] had a significant effect on movement time" and that "the restriction in the extra dimension appeared to affect performance in much the same way as the restriction in width, but to a slightly lesser degree" [5]. Seeing that the target height had a similar logarithmic effect on performance, Crossman suggested a model of the form[3]:

$$T = a + b\,\log_2\left(\frac{D}{W}+1\right) + c\,\log_2\left(\frac{D}{H}+1\right) \quad (2)$$

where a, b, c are constants. Since only two persons participated in his study, it should be considered a pilot experiment.

Crossman's model explicitly separates the difficulty of the amplitude and directional actions by defining two additive indexes of difficulty. Hoffmann & Sheikh do not fully agree with this approach and argue that "only when the target height is less than the natural vertical scatter of hits on the target is there likely to be any effect of vertical constraint" [5]. In other words, the effect of width and height should interact: if one gets very difficult, it should cancel out the effect of the second. This is not true for the Crossman model.

The best known study of bivariate pointing in the HCI community is by MacKenzie & Buxton [9]. Keeping the well-known form of Fitts' law, with a ratio of the distance to be covered and the target "extent", they examined the following five candidates that quantify target extent in 2D (Figure 3):

[3]The model suggested by Crossman [3] does not include the unitary constants within the log term. Since Fitts' law in its Shannon form is more widely accepted in the HCI community, we will use it throughout this paper.

- The minimum of the width and height:

$$ID_{\min(W,H)} = \log_2\left(\frac{D}{\min(W,H)}+1\right) \quad (3)$$

- The "apparent width" W' in the direction of motion:

$$ID_{W'} = \log_2\left(\frac{D}{W'}+1\right) \quad (4)$$

- The sum of target's width and height.
- The product of the target's width and height.
- The target width itself, without taking height into account.

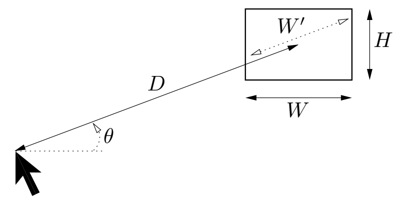


Figure 3: Bivariate target acquisition

Among these five, the $ID_{\min(W,H)}$ model gave the best correlation with experimental data [9], followed by the W' model. The rest were rejected by the authors. The study of MacKenzie & Buxton has been fairly successful as the $ID_{\min}$ model has been used (e.g. [10, 12]) and taught in the past decade. It has an operational simplicity and the advantage of being easily extended to higher dimensions. This model was also separately proposed by Hoffmann & Sheikh [5].

However, all models proposed to date have serious limitations of some kind. For example, the $ID_{W'}$ model that measures target size along movement direction essentially ignores directional constraints. Its validity is also questionable when the "approach angle" is not 0, 45, or 90 degrees, which were the only cases tested in MacKenzie & Buxton's study. But more importantly, such a model does not take account of the limit tasks properly.

Let $ID(D,W,H)$ denote the index of difficulty of the bivariate pointing task with a target of width W and height H lying at a distance from the position of the cursor (Figure 4.a). This index must satisfy the following two properties:

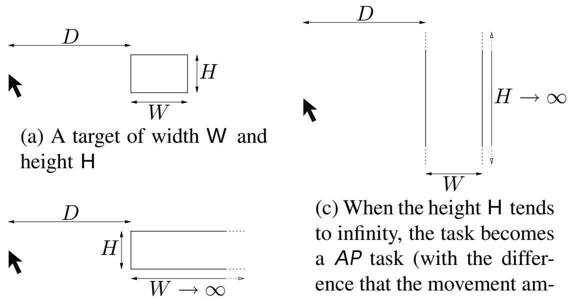
- When W tends to infinity, the limit task is a *DP* task (Figure 4.b). The difficulty of the task should be:

$$\forall D, H; \quad \lim_{W\to\infty} ID(D,W,H) = \log_2\left(\frac{D}{H}+1\right) \quad (5)$$

- When H tends to infinity, the limit task is an *AP* task (Figure 4.c). The difficulty of the task should be:

$$\forall D, W; \quad \lim_{H\to\infty} ID(D,W,H) = \log_2\left(\frac{D}{W}+1\right) \quad (6)$$

Among the models described so far, only the Crossman and $ID_{\min}$ models satisfy these properties. But they are still not satisfactory as they do not properly account for the interaction between target width and height. In the case of Crossman's model, the width and height are fully independent variables and the target height is predicted to contribute to movement time even when it is very large compared to the target



(a) A target of width W and height H

(b) When the width W tends to infinity, the task becomes an *DP* task.

(c) When the height H tends to infinity, the task becomes a *AP* task (with the difference that the movement amplitude is measured to the edge of the target instead of its center).

Figure 4: Bivariate pointing and its two limit tasks

width, which is counter-intuitive. In the case of the $ID_{\min}$ model, the interaction between width and height exists but is very crude: it predicts that the movement time does not depend on H as soon as it is greater than W; vice versa, it is not affected by W as soon as $W > H$. This is difficult to believe and actually inconsistent with available empirical data. For example, as shown in Table 3 in [5], for the same height (e.g. 10), the movement time did increase as W changed (e.g. from 40 to 10). Sheikh & Hoffmann [11] also showed that the difficulty for acquiring a square is indeed greater than for a rectangle, which is not captured by the $ID_{\min}$ model.

MOTIVATION AND DIRECTION OF THE CURRENT STUDY

The unsatisfactory state of affairs in 2D pointing motivated us to conduct a new study to provide a more complete understanding of it. Our study will depart from the prior art in two ways: experimental manipulation and theoretical modeling.

Experimental manipulation

In terms of experimental manipulation, three factors differentiate our study from the prior art. First, knowing that ***DP*** tasks also follow Fitts' law, we view W-dominant ($W < H$) and H-dominant ($H < W$) cases symmetrically and study both as mirror situations. Second, unlike previous works that manipulate absolute H and W values and leave their ratio as a dependent variable, we control the H and W ratio and view it as a basic independent variable. The ratio, which determines the shape of a rectangular target and is hence the fundamental factor to the interaction between H and W, has been an oversight in the literature. Third, we are interested in the impact of the secondary constraint (the greater of H and W) as it approaches the primary constraint. The literature never studied $H : W$ ratios between 1 and 2, and was hence more likely to find only binary all-or-nothing impact. For example, Hoffmann & Sheikh [5] tested the ratios 1, 2, 4, 5, 8, 10, 20, and 40. Our hypothesis is that large ratios will be equivalent to no constraint at all on the low-constraint axis.

Desirable model properties

In terms of quantitative modeling, we start with a set of desirable properties for a bivariate pointing model:

- Scale independency: multiplying D, W and H by a same constant should leave movement time unchanged.
- Limit tasks: the model should regress toward 1D Fitts' law when either H or W tends to infinity.
- Dominance effect: the smaller of H or W should dominate the ID value, with little or smaller impact of the other.
- Duality of H and W: both H and W effects should be contained in the same model and be of similar nature.
- Continuity: the effect of H and W should be continuously represented, rather than stepwise or segmented.

New model candidates

In order to quantify a bivariate task with an univariate index of difficulty, we need to define the appropriate "distance" in a two dimensional space, with the two dimensions determined by W and H. The general mathematical notion of distance we will use for that purpose is the weighted ℓ_p-norm.

The weighted ℓ_p-norm Given a vector $x = (x_1, \ldots, x_n)$, a real number $p \geq 1$ and a set of positive weights $w = (w_1, \ldots, w_n)$, the w-weighted ℓ_p-norm of x is defined by:

$$\|x\|_{p,w} = \left(\sum_{i=1}^{n} w_i |x_i|^p \right)^{1/p} \tag{7}$$

The weighted ℓ_p-norm generalizes common norms, such as:

- the ℓ_1-norm with unitary weights:

$$\|x\|_1 = \sum_{i=1}^{n} |x_i| = |x_1| + \ldots + |x_n| \tag{8}$$

also known as the city-block or Manhattan norm.

- the ℓ_2-norm with unitary weights:

$$\|x\|_2 = \left(\sum_{i=1}^{n} |x_i|^2 \right)^{1/2} = \sqrt{|x_1|^2 + \ldots + |x_n|^2} \tag{9}$$

which is the well-known Euclidean norm.

- the generic ℓ_p-norm ($p > 0$) with unitary weights:

$$\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p} \tag{10}$$

also known as the Hölder or Minkowski norm.

- When p tends to infinity, the value of $\|x\|_p$ reaches asymptotically the maximum value of x_i. The ℓ_∞-norm with unitary weights is hence defined by:

$$\|x\|_\infty = \max_{1 \leq i \leq n} \{|x_i|\} = \max \{|x_1|, \ldots, |x_n|\} \tag{11}$$

ans is also called the Chebyshev, uniform or max norm.

ℓ_p-norm models for bivariate pointing Using the previous notations, we investigate models in the following form:

$$T = a + b \log_2 (\|X\|_{p,w} + 1) \tag{12}$$

where a and b are constants, and X is the constraint vector:

$$X = \left(\frac{D}{W}, \frac{D}{H} \right) \tag{13}$$

Examples of norms of the constraint X are:

$$\|X\|_1 = \frac{D}{W} + \frac{D}{H} \quad (14)$$

$$\|X\|_2 = \sqrt{\left(\frac{D}{W}\right)^2 + \left(\frac{D}{H}\right)^2} \quad (15)$$

$$\|X\|_\infty = \max\left(\frac{D}{W}, \frac{D}{H}\right) \quad (16)$$

Figure 5 presents the constraint space with the X vector as well as the three previous norms.

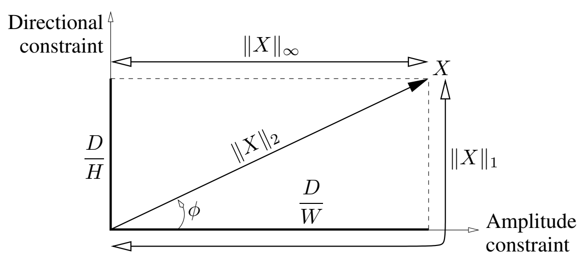


Figure 5: Defining a 2D constraint

Note that the unitary ℓ_∞-norm can be rewritten as:

$$\|X\|_\infty = D \times \max\left(\frac{1}{W}, \frac{1}{H}\right) = \frac{D}{\min(W, H)} \quad (17)$$

which corresponds to the $ID_{\min}$ model described earlier. The ℓ_p-norm model family hence includes the best model to date.

So far we only considered unitary weights. More generally, if we denote by ω and η the weights for the target width and height respectively, the general weighted ℓ_p-norm model is:

$$T = a + b\,\log_2\left(\left[\omega\left(\frac{D}{W}\right)^p + \eta\left(\frac{D}{H}\right)^p\right]^{1/p} + 1\right) \quad (18)$$

Depending on the parameter values, this model satisfies some or all the desirable properties for a model for bivariate pointing. For instance, Equation 18 is continuous in W and H as long as $p < \infty$. When $\omega = \eta = 1$, the model converges towards the 1D version of Fitts' law as soon as W or H tends to infinity. On the other hand, the models with non-unitary weights do not strictly converge towards the 1D Fitts' law. Instead, they converge towards a variant of Fitts' law [1]:

$$T = a + b\,\log_2\left(c\,\frac{D}{E} + 1\right) \quad (19)$$

where c is either ω or η, and the target extent E is W or H.

Model parameterization The models given by Equation 18 include five parameters: a, b, ω, η, and p. This is three more parameters than the standard 1D Fitts' law. Having the number of parameters more than doubled when adding only one degree of freedom to the task (1D to 2D) causes a concern of overfitting. To address this issue, it is important to find parameters that may be superfluous and hence removable from the model. It is possible to remove some parameters by fixing them to specific values. For instance, the norm order p can be fixed to common values — such as 1, 2 or ∞ — or remain a free variable, the best order being estimated. The order p of the models will then be parameterized by $p \in \{1, 2, \infty, \star\}$, where the "$\star$" symbol denotes a parameter to be estimated. Similarly, we consider the cases where none, one or both the weights ω and η are free variables. The number of free weights will be parameterized by $n \in \{0, 1, 2\}$, which correspond to the following (ω, η) combinations: $(1, 1)$, that is both weights are set to unity; $(1, \star)$, that is ω is set to unity and η is a free variable; and $(\star, \star)$, that is both weights are free variables. We always consider a and b free variables. The model of order p with n free weights is denoted by $\mathcal{M}_n^p$. For example, the $ID_{\min}$ model is $\mathcal{M}_0^\infty$, and $\mathcal{M}_1^2$ is the Euclidean model with one free weight:

$$T = a + b\,\log_2\left(\sqrt{\left(\frac{D}{W}\right)^2 + \eta\left(\frac{D}{H}\right)^2} + 1\right) \quad (20)$$

Given the possible combinations of norm orders and weight configurations $(p \times n)$, we will study twelve different models.

EXPERIMENT

Ten people, five female and five male, all right-handed, participated in the experiment.

Apparatus

The experiment was conducted on an IBM® PC equipped with an IBM ScrollPoint™ (model MO09K) and a 19" IBM CRT monitor (model P92, $36\,\text{cm} \times 27\,\text{cm}$ visual area, 1152×864 pixels, 80 dpi resolution). The computer ran Linux™ and used the X Window System™ for graphics. The mouse acceleration was set to its default X settings (acceleration: 2/1; threshold: 4). The experiment was done in full-screen mode, with a black background color.

Procedure and design

The task was to point two rectangular targets on the screen using the mouse (Figure 2). Two targets were shown on the screen, a green and a gray one. As the participant clicked on the green target, the colors of the two targets swapped, as a sign that the pointing action was successful and that the participant had to move to the second target and select it. The participant had to perform such a reciprocal pointing task until the condition changed in size or position of the two targets.

A within-subject factorial design with repeated measures was used. The independent variables were the distance between targets (120, 360, 840 pixels), the minimum target size (8, 24, 48 pixels), the target width and height ratio (1, $1^{1/4}$, $1^{1/2}$, 2, 3, 5, ∞), and the target orientation (horizontal, vertical). For example, we can have: $D = 360$, vertical target ($W < H$), $W = 48$, $1^{1/4}$ ratio and hence $H = 60$. All the tested W-H combinations are summarized in Table 1. There were 120 different combinations of D, W and H in total.

The experiment included two sessions: a practice session, to allow participants to get used to the task and conditions, and a data-collection session, wherein participants tested the 120 different D-W-H combinations in a random order. Within each condition, participants performed 9 trials.

$W = H:$	$W = H = 8$		$W = H = 48$
	$W = H = 24$		$W = H = \infty$
$W < H:$	$W = 8,$	H	$= \{10,\ 12,\ 16,\ 24,\ 40,\ \infty\}$
	$W = 24,$	H	$= \{30,\ 36,\ 48,\ 72,\ 120,\ \infty\}$
	$W = 48,$	H	$= \{60,\ 72,\ 96,\ 144,\ 240,\ \infty\}$
$W > H:$	$H = 8,$	W	$= \{10,\ 12,\ 16,\ 24,\ 40,\ \infty\}$
	$H = 24,$	W	$= \{30,\ 36,\ 48,\ 72,\ 120,\ \infty\}$
	$H = 48,$	W	$= \{60,\ 72,\ 96,\ 144,\ 240,\ \infty\}$

Table 1: Target width & height conditions

Results

Movement time analysis As discussed earlier, the most revealing variable to the characteristics of 2D pointing is the W and H ratio. Figure 6 shows how mean movement time changes as a function of the set of ratios tested in our experiment, while all other factors are balanced.

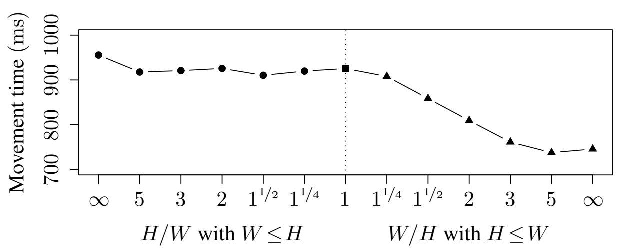


Figure 6: Movement time and width-to-height ratio

Immediately noticeable is that the impact of the $W{:}H$ ratio on movement time depends on whether the amplitude constraint is dominant ($W < H$, left side of the figure) or the directional constraint is dominant ($H < W$, right side).

When the task is W(amplitude)-dominant, one would expect, as H decreases from infinity to the value of W (the H/W ratio decreases from ∞ to 1, from left to center in Figure 6), that the movement time increases due to the increasing secondary (directional) constraint. However, our data reject such an expectation. H plays almost no role when W is dominant. In fact, the mean value *decreases* from $H/W = \infty$ to $H/W = 1$, although a *post hoc* ANOVA test shows this difference is not significant (all P-values greater than .3).

In contrast, when the task is H(direction)-dominant, movement time increases as W decreases from infinity to the value of H (the W/H ratio decreases from infinity to 1, from right to center in Figure 6), just as one would expect from the increasing impact of the secondary (now amplitude) constraint. Such an impact is most obvious between $W/H = 3$ and $W/H = 1$: although there is no signifcant difference between ratio 1 and ratio $1^{1/4}$, all pairwise differences between ratios $1^{1/4}$, $1^{1/2}$, 2 and 3 are significant ($P < .03$). Beyond $W/H = 3$, further relaxing amplitude constraint W did not help much: there is no significant difference between ratios 3 and 5 ($P = .65$), nor between ratios 5 and ∞ ($P = .89$).

Given the different picture of W- vs. H-dominant cases, we separate these two classes in further analysis of other factors.

For W-dominant cases ($W \leq H$), a repeated-measure ANOVA shows that both A ($F_{2,18} = 254,\ p < .0001$) and W ($F_{2,18} = 229,\ p < .0001$) have a significant effect on movement time, but not the H/W ratio ($F_{6,54} = 1.96,\ p = .088$) nor its interaction with W ($F_{12,108} = 1.06,\ p = .4$).

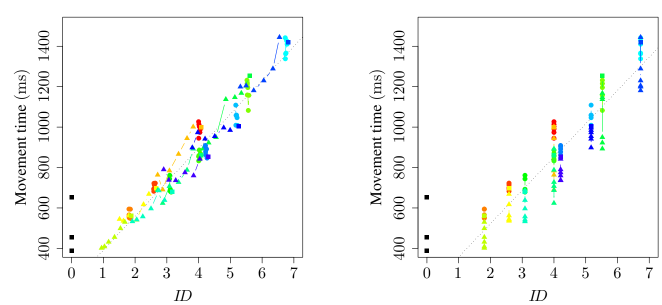
For H-dominant cases ($H \leq W$), a repeated-measure ANOVA shows that both A ($F_{2,18} = 489,\ p < .0001$) and H ($F_{2,18} = 490,\ p < .0001$) as well as the W/H ratio ($F_{6,54} = 43.9, p < .0001$) have a significant effect on movement time. Furthermore, the effect of H depends on the W/H ratio, as indicated by the interaction of the two ($F_{12,108} = 4.02, p < .0001$).

Fit of the model By a least-squares fit method, we estimated the coefficients of the 12 possible models. Table 2 shows the result. As we noticed that the inclusion of conditions with infinite target width or height changed significantly the parameter estimation, the table includes the results for all conditions with finite width and height (99 conditions). The first two columns give the model order and number of free weights; the next five columns give, when applicable, the estimated parameters and the standard error on the estimate; the last column provides the R^2 value for the regression.

From Table 2, we learn that the Euclidean models are slightly better than other finite orders. Furthermore, the Euclidean regression coefficients are equivalent to the best-order models ($p = \star$), suggesting that the Euclidean models are practically optimal. We can also check that the best estimated orders are very close to 2. For all orders, adding one free weight always results in an increased R^2 value. Adding a second weight still improves the fitness to the model, but to a much lesser extent. On the other hand, the standard errors on parameter estimates for two-weight models are much higher than for one-weight models: the slight improvement in accuracy is gained at the detriment of estimation accuracy. Since it is practically more important to have few parameters and accurate estimates, the one-weight models represent the best choice overall, i.e.:

$$T \simeq 223 + 168\ \log_2\left(\sqrt{\left(\frac{D}{W}\right)^2 + \frac{1}{7.3}\left(\frac{D}{H}\right)^2} + 1\right) \quad (21)$$

Figure 7.a plots movement time against the log term. The linear relation is quite strong. As a comparison, Figure 7.b plots movement time as a function of $ID_{\min}$. We clearly see that this model is not as accurate as the Euclidean model: movement times are spread up to $500\,\mathrm{ms}$ for a same ID.



(a) $\mathcal{M}_1^2$ model (b) $\mathcal{M}_0^\infty$ model

Figure 7: Movement time against index of difficulty

p	n	a		b		ω		η		p		R^2
		Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.	
1	0	121	27.7	160	5.62	—	—	—	—	—	—	0.893
1	1	210	20.4	166	4.06	—	—	0.221	0.0466	—	—	0.946
1	2	350	42.3	187	10.6	0.37	0.116	0.0866	0.0302	—	—	0.949
2	0	166	26.3	164	5.77	—	—	—	—	—	—	0.893
2	1	223	18.4	168	4.06	—	—	0.137	0.0335	—	—	0.948
2	2	347	42.4	188	10.4	0.174	0.108	0.0253	0.0163	—	—	0.951
∞	0	193	27.3	165	6.25	—	—	—	—	—	—	0.878
∞	1	240	17.5	168	4.17	—	—	0.422	0.0331	—	—	0.945
∞	2	343	44.9	184	10.4	0.48	0.157	0.205	0.0681	—	—	0.947
$\star$	0	144	38.1	162	6.14	—	—	—	—	1.32	0.53	0.894
$\star$	1	224	19.3	168	4.11	—	—	0.125	0.0768	2.14	0.875	0.948
$\star$	2	347	42.6	188	10.5	0.182	0.151	0.0274	0.0325	1.94	0.752	0.951

Table 2: Summary of modeling results

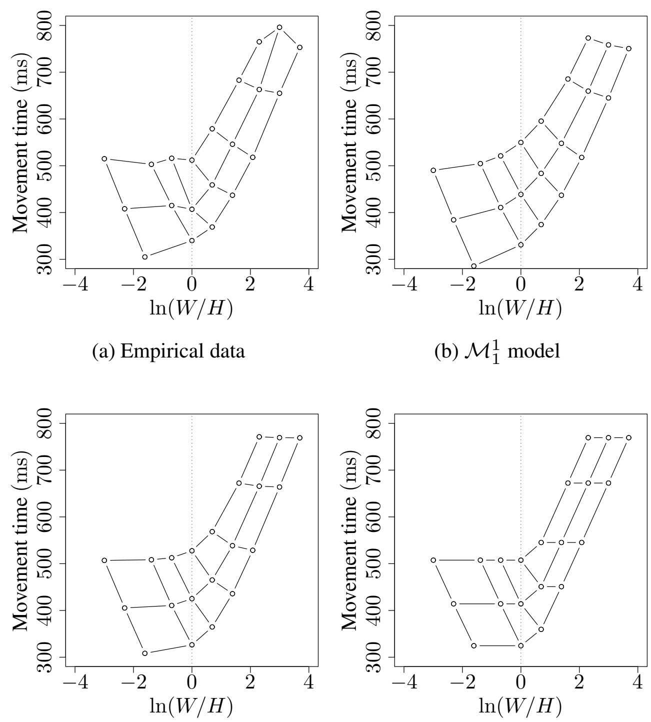
Error analysis For all participants, we recorded 461 errors and 10800 correct trials, that is a 4.3% error rate. This is consistent with standard Fitts' law experiment requirements. There is no significant effect nor interaction for D, W or W/H ratio when $H > W$. When $W > H$, there is a significant effect of H ($F_{2,18} = 3.7,\ P < .05$) and ratio ($F_{6,54} = 3.5,\ P < .006$), without interaction.

REANALYSIS OF THE DATA OF HOFFMANN & SHEIKH

To insure our models and findings are not limited to the current experimental data, this section reanalyzes the data for bivariate pointing provided by Hoffmann & Sheikh [5].

Effects of width and height, and their interaction

To investigate the effects of target width and height, we plot Hoffmann & Sheikh's target acquisition time as a function of $\ln(W/H)$ (Figure 8.a). Each 7-point curve corresponds to one width and each 3-point curve corresponds to one height. The conclusion drawn from Figure 8.a, consistent with what is found from our data, is that the effects of W and H are not symmetrical. If they were, we would see a mirror image around the pivot value $\ln(W/H) = 0$. Instead, H has little effect on pointing time when $W < H$ (flat lines on the left half of the figure) while W has a strong effect when $W > H$. In summary, W has stronger constraining power than H; it is again necessary to distinguish between the cases where W is dominant, and those where H is dominant.



(a) Empirical data (b) $\mathcal{M}_1^1$ model (c) $\mathcal{M}_1^2$ model (d) $\mathcal{M}_1^\infty$ model

Figure 8: Empirical data and model predictions

Fit of the models

Table 3 summarizes the ℓ_p-norm models and their fitness to Hoffmann & Sheik's data. We see that the Euclidean models still provide the best fit. They outperform other models for all n values, including the $ID_{\min}$ model ($\mathcal{M}_0^\infty$ in Table 3) recommended by Hoffmann and Sheik. For all orders, the model precision increases drastically when n changes from 0 to 1. The values of b are around 100 ms; most values of a are very close to 0. When $n = 1$, the estimated value of η is somewhat constant and close to 2; when $n = 2$, the values of ω and η do not seem to follow any obvious pattern.

We can understand the superiority of the one-weight models by performing a quick sensitivity analysis of the parameter estimation (Table 3). For instance, for $(p, n) = (2, 1)$, the standard error on the estimate of η is 0.0397, that is 12% of the estimate; on the other hand, for $(p, n) = (2, 2)$, the standard error is 0.111 for η, that is 152% of the estimate. The estimation becomes 12 times less "accurate" for η when there are two free weights. More generally, changing n from 1 to 2 at any order increases dramatically the uncertainty of the estimation of all parameters. Although the four-parameter models are somewhat more desirable theoretically (they treat W and H symmetry), they behave poorly in practice due to their sensitivity to noise (illconditioning).

p	n	a		b		ω		η		p		R^2
		Estimate	**Std. Err.**	**Estimate**	**Std. Err.**	**Estimate**	**Std. Err.**	**Estimate**	**Std. Err.**	**Estimate**	**Std. Err.**	
1	0	−29.6	27.5	93.4	4.45	—	—	—	—	—	—	0.959
1	1	−90.5	22.7	114	5.36	—	—	0.483	0.0623	—	—	0.984
1	2	157	53.7	134	13.3	0.136	0.0706	0.0634	0.0352	—	—	0.987
2	0	19.9	20.8	89.1	3.5	—	—	—	—	—	—	0.972
2	1	−29.9	12.2	106	2.87	—	—	0.319	0.0397	—	—	0.994
2	2	61.5	89.8	110	5.87	0.233	0.347	0.0731	0.111	—	—	0.994
∞	0	53.8	20.5	84.9	3.51	—	—	—	—	—	—	0.968
∞	1	15.1	14.4	97.6	3.14	—	—	0.659	0.043	—	—	0.989
∞	2	−8.57	268	97.1	6.11	1.22	2.66	0.801	1.75	—	—	0.989
$\star$	0	31.6	26.9	87.8	4.06	—	—	—	—	2.74	1.62	0.972
$\star$	1	−30.9	16.7	107	3.42	—	—	0.323	0.0613	1.97	0.334	0.994
$\star$	2	72.3	86.7	112	7.46	0.204	0.27	0.0677	0.0897	1.86	0.324	0.994

Table 3: Summary of modeling results for Hoffmann & Sheikh's data [5]

Differences in behavior of the tested models

We can see how good the proposed models imitate the behavior of the empirical data (Figure 8.a): the $\mathcal{M}_1^1$ model behaves somewhat like the experimental data for $W > H$, but it is not flat enough when $W < H$ (Figure 8.b); on the contrary, the $\mathcal{M}_1^\infty$ model is totally flat when $W < H$ but does not account for the interaction between W and H when $W > H$ (Figure 8.d); the $\mathcal{M}_1^2$ accommodates both cases and captures more precisely the essence of the data (Figure 8.c).

Conclusions of the analysis of Hoffmann & Sheikh's data

In summary, Hoffmann and Sheikh's results are best represented by the following equation:

$$T \simeq -30 + 106\,\log_2\left(\sqrt{\left(\frac{D}{W}\right)^2 + 0.32\left(\frac{D}{H}\right)^2} + 1\right) \tag{22}$$

Despite the very different settings, Hoffmann and Sheik's results and ours are hence both best represented by the $\mathcal{M}_1^2$ Euclidean model. There are two quantitative differences between the analysis of Hoffmann & Sheikh's data and ours. First, the regression coefficients are much higher with their data than with our data. This is partly due to the fact that we tested many more conditions then Hoffmann and Sheikh and hence have many more points in our regression analysis (99 vs. 21). Second, within the one-weight Euclidean model, the weight of directional constraint in our results is $\eta \simeq 1/7.30$, while it is $\eta \simeq 1/3.14$ in Hoffmann and Sheik's data. This is most likely caused by the different input devices used in the two experiments: While Hoffmann and Sheik used a stylus, which reflects the hand movement more directly, we used today's most common computer input device — the mouse. It is possible that a mouse offers greater directional stability than the stylus, as reflected by the greater η value (i.e. lower weight for the directional term in the Euclidean model). The mouse transfer function often accelerates the X and Y coordinates independently, hence a rapid or large motion in one dimension may not necessarily cause much motion in the other dimension. The directional stability difference between the stylus and the mouse requires further investigation.

DISCUSSION AND CONCLUSION

We have carried out a more complete study on 2D bivariate pointing — pointing with simultaneous amplitude and directional constraint. Previous literature states that it is the smaller of the two dimensions of a 2D target that determines pointing performance. Our investigation showed that these two types of constraint functioned differently. Between amplitude constraint W and directional constraint H, W completely masked the effect of H when $W < H$ (target elongated perpendicularly to cursor movement direction). There was no impact of directional constraint H at all as long as W was smaller than H. In other words, the-smaller-of-the-two rule works when $W < H$. On the other hand, when the amplitude constraint W was greater than the directional constraint H ($W > H$, i.e. the target is elongated in the movement direction), H was not the only determinent of pointing performance and W still had an impact on target acquisition difficulty. The closer W was to H (less elongated), the more difficult that task was, particularly when $W \lesssim 3H$.

The asymmetrical impact of amplitude and directional constraint is plausible in term of the mechanisms underlining the pointing action: while directional error could be continuously regulated in the entire course of pointing movement, amplitude error has to be momentarily controlled at the final "landing".

There are many user interface design implications to the asymmetrical impact of directional and amplitude constraint. For example, since there is no significant performance difference between $W/H = 3$ and higher ratios, over elongated graphical widgets (e.g. buttons) do not offer additional ease to acquire no matter what movement the direction is. For another example, since it is faster to point at a target with larger amplitude tolerance than with larger directional tolerance, when possible graphical widgets should be extended along the more frequent movement direction. The frequently used task bar at the very edge of the desktop interface is a case in point: they should be extended beyond the physical edge of the screen to relax the amplitude contraint, like in the Mac OS desktop (See [2] for a similar analysis). Interestingly, our results also suggest that, from the size-of-constraint point of view, horizontally enlongated widgets, which are often due to the labeling in English words, should be placed on the left or right rather than the top or bottom edge of the desktop interface. However, the average horizontal movement distance is somewhat longer due to the landscape display geometry in most computers. The quantitative difference can be calculated by the formal models with specific distance and 2D constraint values.

The current work also provides a foundation to answer the question of the distractor effect in goal crossing [2]. Indeed, we have previously shown that goal crossing takes the same or less time than their pointing counterpart, with the same distance and size constraint. However, it was not clear if this would still be true if there was a distractor goal before the target goal (Figure 9). With results of this study, we know that placing the cursor (or the stylus tip) between the distractor and the target goal is in fact a 2D pointing task that can be modeled by Equation 23. This means that we can precisely calculate the impact of a distractor as a function of its distance to the target goal.

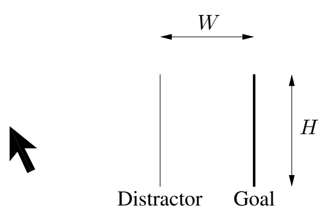


Figure 9: Distractor effect in goal crossing

With regard to formal modeling of 2D pointing tasks, we found it is possible to fit movement time in all 2D cases with a three-parameter Euclidean model (Equation 23). The unequal impact of H and W could be incorporated in such a model by weighting the effect of directional constraint H with η. Depending on factors such as the input device used η typically is between 3 and 7. Its value can be viewed as an indication of the directional stability of the input device. As an potential application, η (or the ratio η/ω if ω is not unitary) may be used as another comparison metric for evaluating input devices, in the spirit of the ISO 9241-9 standard [6].

The validity of the one weight Euclidean model exceeds that of the prior art. It not only fits the data collected in our current experiment, including $W\!:\!H$ ratios close to 1, but also data published from a previous experiment [5]. In contrast, previous models, such as the $ID_{\min}$ model, could not explain our empirical findings, particularly the fact that W influences movement time even if it is greater than H. With the $ID_{\min}$ model, W's influence should vanish if $H < W$. Also note the non-linear nature of the Euclidean model: while the weight of the directional term is 3 to 7 times less than the amplitude term, it should not be interpreted that the directional tolerance counts for $1/3$ to $1/7$ of the amplitude constraint of the same size.

Based on these analyses, it is apparent that the Euclidean model with one weight ($\mathcal{M}_1^2$ model, Equation 23) is the best model among all the candidates: it has greater predictive power with fewer number of parameters. In summary, one may remember that bivariate pointing is closely modeled by:

$$T = a + b\,\log_2\left(\sqrt{\left(\frac{D}{W}\right)^2 + \eta\left(\frac{D}{H}\right)^2} + 1\right) \qquad (23)$$

where a varies approximately in the range of $[-50, 200]$, b in $[100, 170]$, and η in $[1/7, 1/3]$.

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